Table 7-2 NTU relations for heat exchangers.

$C = C_{min}/C_{max}$ ϵ = effectiveness	$N = NTU = UA/C_{min}$
Flow geometry	**Relation**
Double pipe:	
Parallel flow	$N = \dfrac{-\ln[1-(1+C)\epsilon]}{1+C}$
Counterflow	$N = \dfrac{1}{C-1}\ln\left(\dfrac{\epsilon-1}{C\epsilon-1}\right)$
Counterflow, $C = 1$	$N = \dfrac{\epsilon}{1-\epsilon}$
Cross flow:	
C_{max} mixed, C_{min} unmixed	$N = -\ln\left[1+\dfrac{1}{C}\ln(1-C\epsilon)\right]$
C_{max} unmixed, C_{min} mixed	$N = \dfrac{-1}{C}\ln[1+C\ln(1-\epsilon)]$
Shell and tube:	
One shell pass, 2, 4, 6, tube passes	$N = -(1+C^2)^{-1/2} \times \ln\left[\dfrac{2/\epsilon-1-C-(1+C^2)^{1/2}}{2/\epsilon-1-C+(1+C^2)^{1/2}}\right]$
All exchangers, $C = 0$	$N = -\ln(1-\epsilon)$

Off-Design Calculation of Exchanger in Example 7-1

Example 7-5

The heat exchanger of Example 7-1 is used for heating water as described in the example. Using the same entering-fluid temperatures, calculate the exit water temperature when only 40 kg/min of water is heated but the same quantity of oil is used. Also calculate the total heat transfer under these new conditions.

Calculation of Heat-Exchanger Size from Known Temperatures

Example 7-1

Water at the rate of 68 kg/min is heated from 35 to 75°C by an oil having a specific heat of 1.9 kJ/kg · °C. The fluids are used in a counterflow double-pipe heat exchanger, and the oil enters the exchanger at 110°C and leaves at 75°C. The overall heat-transfer coefficient is 320 W/m^2 · °C. Calculate the heat-exchanger area.

Solution

The flow rate of oil is calculated from the energy balance for the original problem:

$$\dot{m}_h c_h \,\Delta T_h = \dot{m}_c c_c \,\Delta T_c \qquad [a]$$

$$\dot{m}_h = \frac{(68)(4180)(75-35)}{(1900)(110-75)} = 170.97 \text{ kg/min}$$

The capacity rates for the new conditions are now calculated as

$$\dot{m}_h c_h = \frac{170.97}{60}(1900) = 5414\ \text{W}/^\circ\text{C}$$

$$\dot{m}_c c_c = \frac{40}{60}(4180) = 2787\ \text{W}/^\circ\text{C}$$

so that the water (cold fluid) is the minimum fluid, and

$$\frac{C_{\min}}{C_{\max}} = \frac{2787}{5414} = 0.515$$

$$\text{NTU}_{\max} = \frac{UA}{C_{\min}} = \frac{(320)(15.82)}{2787} = 1.816 \qquad [b]$$

where the area of 15.82 m^2 is taken from **Example 7-1**. From **Figure 7-14** or **Table 7-1** the effectiveness is

$$\epsilon = \frac{\Delta T(minimum\ fluid)}{Maximum\ temperature\ difference\ in\ heat\ exchanger} = \frac{\Delta T_{cold}}{\Delta T_{max}} = \frac{\Delta T_{cold}}{110-35} = 0.744$$

$$\Delta T_{\text{cold}} = 55.8^\circ\text{C}$$

and the exit water temperature is

$$T_{w,\text{exit}} = 35 + 55.8 = 90.8^\circ\text{C}$$

The total heat transfer under the new flow conditions is calculated as

$$q = \dot{m}_c c_c \Delta T_c = \frac{40}{60}(4180)(55.8) = 155.5\ \text{kW} \qquad [5.29 \times 10^5\ \text{Btu/h}] \qquad [d]$$

Notice that although the flow rate has been reduced by 41 percent (68 to 40 kg/min), the heat transfer is reduced by only 18 percent (189.5 to 155.5 kW) because the exchanger is more effective at the lower flow rate.

Example 7-6 Cross-Flow Exchanger with Both Fluids Unmixed

A finned-tube heat exchanger like that shown in Figure 7-4 is used to heat 5000 ft^3/min [2.36 m^3/s] of air at 1 atm from 60 to 85°F (15.55 to 29.44°C). Hot water enters the tubes at 180°F [82.22°C], and the air flows across the tubes, producing an average overall heat-transfer coefficient of 40 Btu/h · ft^2 · °F [227 W/m^2 · °C]. The total surface area of the exchanger is 100 ft^2 [9.29 m^2]. Calculate the exit water temperature and the heat-transfer rate.

Solution

The heat transfer is calculated from the energy balance on the air. First, the inlet air density is

$$\rho = \frac{p}{RT} = \frac{1.0132 \times 10^5}{(287)(288.7)} = 1.223\ \text{kg/m}^3$$

so the mass flow of air (the cold fluid) is

$$\dot{m}_c = (2.36)(1.223) = 2.887\ \text{kg/s}$$

The heat transfer is then

$$q = \dot{m}_c c_c \,\Delta T_c = (2.887)(1006)(29.44 - 15.55)$$
$$= 40.34 \text{ kW} \quad [1.38 \times 10^5 \text{ Btu/h}] \qquad [a]$$

From the statement of the problem we do not know whether the air or water is the minimum fluid. If the air is the minimum fluid, we may immediately calculate NTU and use **Figure 7-16** to determine the water-flow rate and hence the exit water temperature. If the water is the minimum fluid, a trial-and-error procedure must be used with **Figure 7-16** or **Table 7-1**. We assume that the air is the minimum fluid and then check our assumption. Then

$$\dot{m}_c c_c = (2.887)(1006) = 2904 \text{ W/°C}$$

and

$$\text{NTU}_{\text{max}} = \frac{UA}{C_{\text{min}}} = \frac{(227)(9.29)}{2904} = 0.726$$

and the effectiveness based on the air as the minimum fluid is

$$\epsilon \frac{\Delta T_{\text{air}}}{\Delta T_{\text{max}}} = \frac{29.44 - 15.55}{82.22 - 15.55} = 0.208 \qquad [b]$$

Entering **Figure 7-16** we are unable to match these quantities with the curves. This requires that the hot fluid be the minimum. We must therefore assume values for the water-flow rate until we are able to match the performance as given by **Figure 7-16** or **Table 7-1**. We first note that

$$C_{\text{max}} = \dot{m}_c c_c = 2904 \text{ W/°C} \qquad [c]$$

$$\mathrm{NTU_{max}} = \frac{UA}{C_{min}} \qquad [d]$$

$$\epsilon = \frac{\Delta T_h}{\Delta T_{max}} = \frac{\Delta T_h}{82.22 - 15.55} \qquad [e]$$

$$\Delta T_h = \frac{4.034 \times 10^4}{C_{min}} = \frac{4.034 \times 10^4}{C_h} \qquad [f]$$

The iterations are:

				ϵ	
$\frac{C_{min}}{C_{max}}$	$C_{min} = \dot{m}_h c_h$	$\mathrm{NTU_{max}}$	ΔT_h	From Figure 7-16 or Table 7-1	Calculated from Equation (*e*)
0.5	1452	1.452	27.78	0.65	0.417
0.25	726	2.905	55.56	0.89	0.833
0.22	639	3.301	63.13	0.92	0.947

We thus estimate the water-flow rate as about

$$\dot{m}_h c_h = 660 \text{ W/°C}$$

and

$$\dot{m}_h = \frac{660}{4180} = 0.158 \text{ kg/s}$$

The exit water temperature is accordingly

$$T_{w,exit} = 82.22 - \frac{4.034 \times 10^4}{660} = 21.1\text{°C}$$

Alternatively, Equations (*c, d, e, f*) may be rearranged to give

$$N = 0.7762/C \qquad [g]$$

$$\epsilon = 0.22084/C \qquad [h]$$

where N and C are defined as in Table 7-1 The appropriate effectiveness equation from Table 7-1 (cross flow, both fluids unmixed) is

$$\epsilon = 1 - \exp\{[\exp(-NCn) - 1]/Cn\} \qquad [i]$$

where $n = N^{-0.22}$

Substituting Equations (g) and (h) in Equation (i) gives a single equation in terms of the capacity ratio C, which may be solved numerically to yield

$$C = 0.23$$

The value of C_{min} is then

$$C_{min} = 2904 \times C = (2904)(0.23) = 668 \text{ W/°C}$$

Or, a slightly different value from the above iteration. The resulting exit water temperature is thus

$$T_{w,exit} = 82.22 - 40{,}340/668 = 21.8\text{°C}$$

Example 7-7

Shell-and-Tube Exchanger as Air Heater

Hot oil at 100°C is used to heat air in a shell-and-tube heat exchanger. The oil makes six tube passes and the air makes one shell pass; 2.0 kg/s of air are to be heated from 20 to 80°C. The specific heat of the oil is 2100 J/kg · °C, and its flow rate is 3.0 kg/s. Calculate the area required for the heat exchanger for $U = 200$ W/m^2 · °C.

Solution

The basic energy balance is

$$\dot{m}_o c_o \,\Delta T_o = \dot{m}_a c_{pa}\, \Delta T_a$$

or

$$(3.0)(2100)(100 - T_{oe}) = (2.0)(1009)(80 - 20)$$
$$T_{oe} = 80.78°\text{C}$$

We have

$$\dot{m}_h c_h = (3.0)(2100) = 6300 \text{ W/°C}$$
$$\dot{m}_c c_c = (2.0)(1009) = 2018 \text{ W/°C}$$

so the air is the minimum fluid and

$$C = \frac{C_{\min}}{C_{\max}} = \frac{2018}{6300} = 0.3203$$

The effectiveness is

$$\epsilon = \frac{\Delta T_c}{\Delta T_{\max}} = \frac{80 - 20}{100 - 20} = 0.75$$

Now, we may use either **Figure 7-17** or the analytical relation from **Table 7-2** to obtain NTU. For this problem we choose to use the table.

$$N = -(1 + 0.3203^2)^{-1/2} \ln\left[\frac{2/0.75 - 1 - 0.3203 - (1 + 0.3203^2)^{1/2}}{2/0.75 - 1 - 0.3203 + (1 + 0.3203^2)^{1/2}}\right]$$
$$= 1.99$$

Now, with $U = 200$ we calculate the area as

$$A = \text{NTU}\frac{C_{\min}}{U} = \frac{(1.99)(2018)}{200} = 20.09 \text{ m}^2$$

Ammonia Condenser — Example 7-8

A shell-and-tube heat exchanger is used as an ammonia condenser with ammonia vapor entering the shell at 50°C as a saturated vapor. Water enters the single-pass tube arrangement at 20°C and the total heat transfer required is 200 kW. The overall heat-transfer coefficient is 1000 W/m^2 · °C. Determine the area to achieve a heat exchanger effectiveness of 60 percent with an exit water temperature of 40°C. What percent reduction in heat transfer would result if the water flow is reduced in half while keeping the heat exchanger area and U the same?

Solution

The mass flow can be calculated from the heat transfer with

$$q = 200 \text{ kW} = \dot{m}_w c_w \, \Delta T_w$$

so

$$\dot{m}_w = \frac{200}{(4.18)(40-20)} = 2.39 \text{ kg/s}$$

Because this is a condenser the water is the minimum fluid and

$$C_{\min} = \dot{m}_w c_w = (2.39)(4.18) = 10 \text{ kW/°C}$$

The value of NTU is obtained from the last entry of Table 7-2, with $\epsilon = 0.6$:

$$N = -\ln(1-\epsilon) = -\ln(1-0.6) = 0.916$$

so that the area is calculated as

$$A = \frac{C_{\min} N}{U} = \frac{(10{,}000)(0.916)}{1000} = 9.16 \text{ m}^2$$

When the flow rate is reduced in half the new value of NTU is

$$N = \frac{UA}{C_{\min}} = \frac{(1000)(9.16)}{(10{,}000/2)} = 1.832$$

And the effectiveness is computed from the last entry of Table 7-1:

$$\epsilon = 1 - e^{-N} = 0.84$$

The new water temperature difference is computed as

$$\Delta T_w = \epsilon(\Delta T_{\max}) = (0.84)(50-20) = 25.2°\text{C}$$

so the new heat transfer is

$$q = C_{\min} \, \Delta T_w = \left(\frac{10{,}000}{2}\right)(25.2) = 126 \text{ kW}$$

So, by reducing the flow rate in half we have lowered the heat transfer from 200 to 126 kW, or by 37 percent.

Example 7.9

The condenser of a large steam power plant is a heat exchanger in which steam is condensed to liquid water. Assume the condenser to be a *shell-and-tube* heat exchanger consisting of a single shell and 30,000 tubes, each executing two passes. The tubes are of thin wall construction with *D* _ 25 mm, and steam condenses on their outer surface with an associated convection coefficient of *ho* _ 11,000 W/m2 _ K. The heat transfer rate that must be effected by the exchanger is $q = 2 * 10^9$ W, and this is accomplished by passing cooling water through the tubes at a rate of 3 * 10^4 kg/s (the flow rate per tube is therefore 1 kg/s). The water enters at 20°C, while the steam condenses at 50°C. What is the temperature of the cooling water emerging from the condenser? What is the required tube length *L* per pass?

Solution:

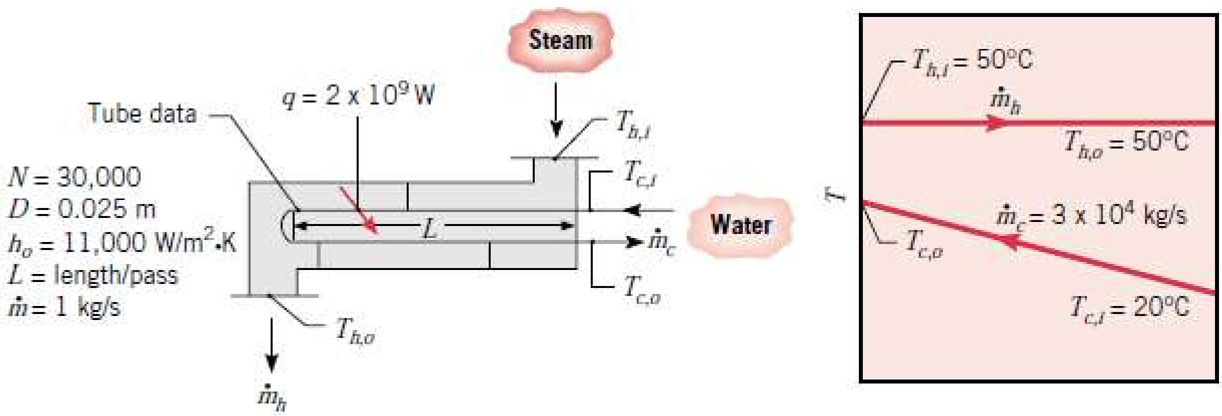
Known: Heat exchanger consisting of single shell and 30,000 tubes with two passes each.

Find:

1. Outlet temperature of the cooling water.

2. Tube length per pass to achieve required heat transfer.

Schematic:



Assumptions:

1. Negligible heat transfer between exchanger and surroundings and negligible kinetic and potential energy changes.
2. Tube internal flow and thermal conditions fully developed.
3. Negligible thermal resistance of tube material and fouling effects.
4. Constant properties.

Properties: , water (assume $\overline{T}_c \approx 27°\text{C} = 300\ \text{K}$): $\rho = 997\ \text{kg/m}^3$, $c_p = 4179\ \text{J/kg}\cdot\text{K}$, $\mu = 855 \times 10^{-6}\ \text{N}\cdot\text{s/m}^2$, $k = 0.613\ \text{W/m}\cdot\text{K}$, $Pr = 5.83$.

Analysis:

1. The cooling water outlet temperature may be obtained from the overall energy balance, Equation 11.7b. Accordingly,

$$T_{c,o} = T_{c,i} + \frac{q}{\dot{m}_c c_{p,c}} = 20°\text{C} + \frac{2 \times 10^9\ \text{W}}{3 \times 10^4\ \text{kg/s} \times 4179\ \text{J/kg}\cdot\text{K}}$$

$$T_{c,o} = 36.0°\text{C}$$ ◁

2. The problem may be classified as one requiring a *heat exchanger design calculation*. First, we determine the overall heat transfer coefficient for use in the NTU method.

$$U = \frac{1}{(1/h_i) + (1/h_o)}$$

where h_i may be estimated from an internal flow correlation. With

$$Re_D = \frac{4\dot{m}}{\pi D\mu} = \frac{4 \times 1 \text{ kg/s}}{\pi(0.025 \text{ m})855 \times 10^{-6} \text{ N} \cdot \text{s/m}^2} = 59{,}567$$

the flow is turbulent and from Equation 8.60

$$Nu_D = 0.023\, Re_D^{4/5} Pr^{0.4} = 0.023(59{,}567)^{0.8}(5.83)^{0.4} = 308$$

Hence

$$h_i = Nu_D \frac{k}{D} = 308 \frac{0.613 \text{ W/m} \cdot \text{K}}{0.025 \text{ m}} = 7543 \text{ W/m}^2 \cdot \text{K}$$

$$U = \frac{1}{[(1/7543) + (1/11{,}000)] \text{ m}^2 \cdot \text{K/W}} = 4474 \text{ W/m}^2 \cdot \text{K}$$

Using the design calculation methodology, we note that

$$C_h = C_{max} = \infty$$

and

$$C_{min} = \dot{m}_c c_{p,c} = 3 \times 10^4 \text{ kg/s} \times 4179 \text{ J/kg} \cdot \text{K} = 1.25 \times 10^8 \text{ W/K}$$

from which

$$\frac{C_{min}}{C_{max}} = C_r = 0$$

The maximum possible heat transfer rate is

$$q_{max} = C_{min}(T_{h,i} - T_{c,i}) = 1.25 \times 10^8 \text{ W/K} \times (50 - 20) \text{ K} = 3.76 \times 10^9 \text{ W}$$

from which

$$\varepsilon = \frac{q}{q_{max}} = \frac{2 \times 10^9 \text{ W}}{3.76 \times 10^9 \text{ W}} = 0.532$$

From Equation 11.35b or Figure 11.12, we find NTU = 0.759. From Equation 11.24, it follows that the tube length per pass is

$$L = \frac{\text{NTU} \cdot C_{min}}{U(N2\pi D)} = \frac{0.759 \times 1.25 \times 10^8 \text{ W/K}}{4474 \text{ W/m}^2 \cdot \text{K}\,(30{,}000 \times 2 \times \pi \times 0.025 \text{ m})} = 4.51 \text{ m} \quad \triangleleft$$

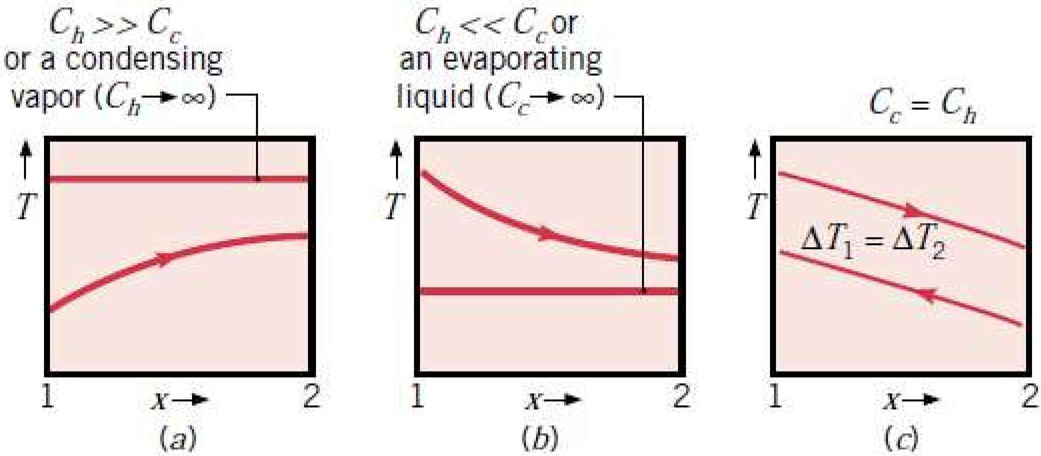


FIGURE 11.9 Special heat exchanger conditions. (*a*) $C_h \gg C_c$ or a condensing vapor. (*b*) An evaporating liquid or $C_h \ll C_c$. (*c*) A counterflow heat exchanger with equivalent fluid heat capacities ($C_h = C_c$).